

State Equations

The behavior of a clocked sequential circuit can be described algebraically by means of state equations. A *state equation* (also called a *transition equation*) specifies the next state as a function of the present state and inputs. Consider the sequential circuit shown in **Fig. 5.15**. We will later show that it acts as a 0-detector by asserting its output when a 0 is detected in a stream of 1's. It consists of two *D* flip-flops *A* and *B*, an input *x* and an output *y*. Since the *D* input of a flip-flop determines the value of the next state (i.e., the state reached after the clock transition), it is possible to write a set of state equations directly from the logic diagram in **Fig. 5.15**:

1 Here the + symbol denotes the logical OR operator; the logical AND operator is not shown explicitly (e.g., Bx is the result of ANDing B with x).

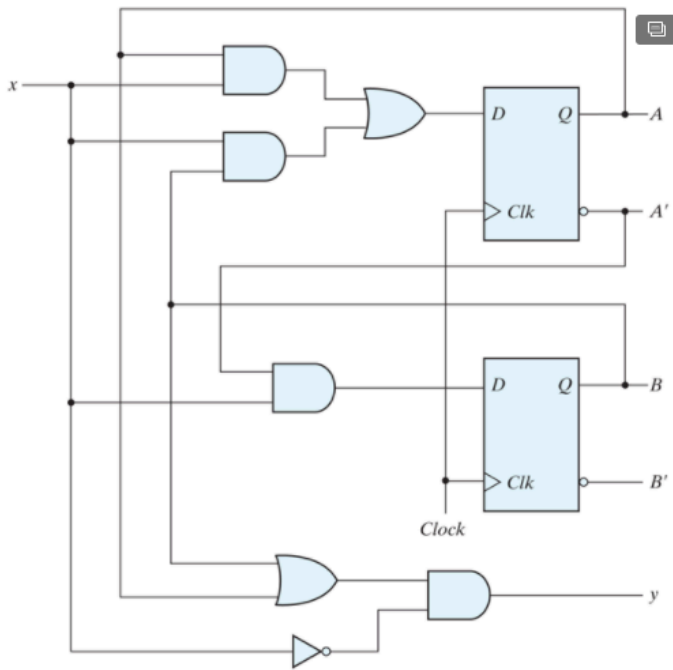


FIGURE 5.15
Example of sequential circuit

$$A(t+1) = A(t)x(t) + B(t)x(t)$$

$$B(t+1) = A(t)x(t)$$

A state equation is an algebraic expression that specifies the condition for a flip-flop state transition. The left side of the equation, with $(t+1)$, denotes the next state of the flip-flop one clock edge later. The right side of the equation is a Boolean expression that specifies the present state and input conditions that make the next state equal to 1. Since all the variables in the Boolean expressions are a function of the present state, we can omit the designation (t) after each variable for convenience and can express the state equations in the more compact form

$$A(t+1) = Ax + Bx$$

$$B(t+1) = Ax$$

The Boolean expressions for the state equations can be derived directly from the gates that form the combinational circuit part of the sequential circuit, since the *D* values of the combinational circuit determine the next state. Similarly, the present-state value of the output can be expressed algebraically as

$$y(t) = [A(t) + B(t)]x'(t)$$

By removing the symbol (t) for the present state, we obtain the output Boolean equation:

$$y = (A + B)x'$$