CL=CSCI 160

CLASS 3

1–12. Perform the following binary multiplications:

(a) 1101×1011 **(b)** 0101×1010 **(c)** 100111×011011

(a)

```
      1101 x
      1011
  --------
      1101
     1101
   1101
  --------
  10001111
```

(c)

```
      100111 x
       11011
  -----------
      100111
     100111
   100111
  100111
  -----------
  10000011101
```

1–14. A limited number system uses base 12. There are at most four integer digits. The weights of the digits are 12^3, 12^2, 12, and 1. Special names are given to the weights as follows: 12 = 1 dozen, 12^2 = 1 gross, and 12^3 = 1 great gross.

(a) How many beverage cans are in 6 great gross + 8 gross + 7 dozen + 4?
(b) Find the representation in base 12 for 7569_{10} beverage cans.

$12^2 = 144$; $12^3 = 1728$

(a) $6 \times \overbrace{1728}^{=12^3} + 8 \times \overbrace{144}^{=12^2} + 7 \times 12 + 4 = \ldots$

(b) $7569_{10} = 4 \text{ gr. gross}^{1728} + 4 \text{ gross}^{144} + 6 \text{ doz}^{12} + 9 =$

7569
6912
= 657
576
= 81
72
9

$= \boxed{4\ 4\ 6\ 9_{\text{base } 12}}$

1–16. *In each of the following cases, determine the radix r:
(a) $(BEE)_r = (2699)_{10}$ **(b)** $(365)_r = (194)_{10}$

(a) $B \cdot r^2 + E \cdot r + E = 2699$ (B = 11, E = 14, E = 14)

$Br^2 + Er = 2699 - 14 = 2685$

alternative 1 (Note: r is a divisor of 2685: factorize it!)

$r(Br + E) = 2685 = 179 \times 3 \times 5$ (3×5: base) → $15 = r$

alternative 2 (Use quadratic formula)

$11r^2 + 14r - 2685 = 0$ Use formula to solve quadratic equations to get the result, but the coefficients are large!

$r_{1,2} = 15$ / $-$ something

(b) Solve: $3r^2 + 6r + 5 - 194 = 0$ ($5 - 194 = -189$)

Use same formula to solve quadratic equations to get the result:

Result $r = 7$

(r-1)'s Complement-Representation

r = base   Very important: How are our numbers stored?

$$N = \underbrace{\overbrace{\qquad\qquad}^{\text{integer part}}}_{n} \bullet \underbrace{\overbrace{\qquad\qquad}^{\text{fraction part}}}_{m}$$

n, m = number of locations

Definition

r = 2   (N is a binary number!)

How did you compute N_{1c} = 1's complement of N? By swapping 0 <-> 1, which is equivalent to:

$$N_{1c} = \underbrace{1 \ldots\ldots 1}_{n}.\underbrace{1 \ldots\ldots 1}_{m} - N = 1\underbrace{0 \ldots\ldots 0}_{n}.\underbrace{0 \ldots\ldots 0}_{m} - 0.\underbrace{0 \ldots\ldots 01}_{m} - N = 2^{n} - 2^{-m} - N$$

Base r:

$$N_{(r-1)c} = r^{n} - r^{-m} - N$$

Algorithm for subtracting two numbers using only addition and the (r-1)'s complement

To perform $M - N$ do:

1) $M + N_{(r-1)c}$

2)

a) If there is an e. a. c. (= end-around-carry = overflow), then add it to the l. s. d. (= least significant digit) of result from 1). Stop.

b) If there is no e. a. c., then the result is negative, and is obtained by taking the (r-1)'s complement of what we obtained at 1); in other words, we compute:

$$-(\underbrace{M + N_{(r-1)c}}_{\text{Result from 1)}})_{(r-1)c}$$

Stop.

Example in binary + HW assigned

Alg (M-N): 1) $M + N_{(r-1)c}$ 2) a) e.a.c. -> add it to l.s.d.
b) no e.a.c. -> compute -(r-1)'s compl. of 1)

Ex-binary r = 2 Suppose n = 4, m = 2

2-a) Path: i)

Suppose: M: 1101.10
N: 1011.01

Apply algorithm to get M - N:

```
         1  1   11
1) M:     1101.10  +
   N1C:   0100.10
         ---------
e.a.c.= (1)0010.00  +
->case2-a)      1
         ---------
          0010.01
```

Result: **0010.01**

2-b) Path: ii) Swap M <-> N, to get to case 2-b)

M: 1011.01
N: 1101.10

Apply algorithm to get M - N:

```
           1     1
1) M:     1011.01  +
   N1C:   0010.01
         ---------
          1101.10   no e.a.c. ->case2-b)
-> -1's compl.:
```

Result: **- 0010.01** as expected

HW (more to follow):

Why does this algorithm work?

Why does this algorithm work?

Hint

We know: $N = \underbrace{\qquad\qquad}_{n}^{\text{integer part}} \bullet \underbrace{\qquad\qquad}_{m}^{\text{fraction part}}$

$N_{(r-1)c} = r^n - r^{-m} - N$

1) $M + N_{(r-1)c} = M + r^n - r^{-m} - N$

2) Whether we follow a) or b) depends on the presence of an e. a. c.

What is the magnitude of the e.a.c.? It's an 'overflow':

e.a.c : $1\underbrace{0 \ldots\ldots 0}_{n}$ = r^n

On **branch a)** there is an e. a. c., which means we have:

$\underbrace{M + r^n - r^{-m} - N}_{\text{from 1)}} \geq \underbrace{r^n}_{\text{e.a.c.}}$ <--> $M - N \geq r^{-m}$ <-->

r^{-m} =smallest positive number in our representation

<--> $M - N > 0$ or $M > N$

It also means, that the case when $M - N = 0$ will take branch b), which means that 0 will be expressed as -0 by this Alg.

Continue justifying the computations in the branches 2-a) and 2-b) as HW.

3-A Do the following conversion problems:

(a) Convert decimal 34.4375 to binary.

(b) Calculate the binary equivalent of 1/3 out to 8 places. Then convert from binary to decimal. How close is the result to 1/3?

(c) Convert the binary result in (b) into hexadecimal. Then convert the result to decimal. Is the answer the same?

3-B **Determine the value of base x if $(211)_x = (152)_8$.**

3-C Noting that $3^2 = 9$, formulate a simple procedure for converting base-3 numbers directly to base-9. Use the procedure to convert $(211020110222\,0112)_3$ to base 9.

3-D The solutions to the quadratic equation

$$x^2 - 11x + 22 = 0$$

are x = 3 and x = 6.

Determine the base of the numbers in the equation.

3-E Convert the hexadecimal number 68BE to binary, and then convert it from binary to octal and then to base 32.