CL=CSCI 160

CLASS 7

HW 6.2-solution:

Prove Properties:

8) $x + 1 = 1$

$x \cdot 0 = 0 \;\; \forall\, x \in B$

Proof

$$x + \underbrace{1}_{x + x'} \overset{(5)}{=} \overbrace{x + x}^{x} + x' \overset{(7)}{=} x + x' \overset{(5)}{=} 1$$

9) $x'' = x, \forall\, x \in B$ <--> Prove: $(x')' = x$

Proof We need to prove that x is the complement of x'. How?

What is the complement? The complement is defined by (5).

By (5), x' is the complement of x if the equalities in (5) hold.

This is (5):

5) $\forall\, x \in B, \exists\, x' \in B,$
$x + x' = 1$ and
$x x' = 0$

Now $x' \in B$, and by (5) has a complement in B.

Is x its complement? Use (5). Substitute x for x' and x' for x:

$$\overset{\text{element}}{x'} + \overset{\text{its compl.}}{x} = 1 \quad \text{and}$$
$$x' \cdot x = 0$$

Do these equalities hold?

Yes, by (3) and (5). [(3) = commutativity i.e. $x' + x = x + x' = 1$ and $x' \cdot x = x \cdot x' = 0$]

$\boxed{\text{So } x \text{ is the complement of } x'.}$

HW 6.2-solution (Continued):

10) DeMorgan's law:

$$(x + y)' = x'\,y'$$
$$(x\,y)' = x' + y' \quad \forall\, x, y \in B$$

Proof

By duality we need to prove only the first equality: $(x + y)' = x'y'$

We use the same reasoning as for the previous property, 9).

That is, we use the axiom that defines the complement, namely 5):

5) $\forall\, x \in B, \exists\, x' \in B,$
$x + x' = 1$ and
$x\,x' = 0$

In the first equality, which is the element and which do we have to prove is the complement?

Element = $x + y$, and its complement = $x'y'$.

Substitute in 5). We need to prove:

$$x + y + x'\,y' = 1$$
$$(x + y) \cdot (x'\,y') = 0$$

HW 7.1 - assigned: If not done yet, continue by proving that the two equalities above hold!

HW 7.2-assigned:

Prove:

$$(x+y)(y+z)(z+x') = (x+y)(z+x')$$

HW 7.3-assigned (4 exercises):

2–2. *Prove the identity of each of the following Boolean equations, using algebraic manipulation:

(a) $\overline{X}\,\overline{Y} + \overline{X}Y + XY = \overline{X} + Y$

(b) $\overline{A}B + \overline{B}\,\overline{C} + AB + \overline{B}C = 1$

(c) $Y + \overline{X}Z + X\overline{Y} = X + Y + Z$

(d) $\overline{X}\,\overline{Y} + \overline{Y}Z + XZ + XY + Y\overline{Z} = \overline{X}\,\overline{Y} + XZ + Y\overline{Z}$

2–4. +Given that $A \cdot B = 0$ and $A + B = 1$, use algebraic manipulation to prove that

$$(A + C)\cdot(\overline{A} + B)\cdot(B + C) = B \cdot C$$

2–8. Using DeMorgan's theorem, express the function

$$F = A\overline{B}C + \overline{A}\,\overline{C} + AB$$

(a) with only OR and complement operations

(b) with only AND and complement operations.

2–9. *Find the complement of the following expressions:

(a) $A\overline{B} + \overline{A}B$

(b) $(\overline{V}W + X)Y + \overline{Z}$

(c) $WX(\overline{Y}Z + Y\overline{Z}) + \overline{W}\,\overline{X}(\overline{Y} + Z)(Y + \overline{Z})$

(d) $(A + \overline{B} + C)(\overline{A}\,\overline{B} + C)(A + \overline{B}\,\overline{C})$