# Logic Gates

## CLASS 9

**HW 6.2-solution** (Continued):

**10) DeMorgan's law:**

$$(x+y)' = x'y'$$

$$(xy)' = x' + y' \quad \forall\, x, y \in B$$

## Proof

By duality we need to prove only the first equality: $(x + y)' = x'y'$

We use the same reasoning as for the previous property, 9).

That is, we use the axiom that defines the complement, namely 5):

In the first equality, which is the element and which do we have to prove is the complement?

Element = $x + y$, and its complement = $x'y'$.

Substitute in 5). We need to prove:

$$x + y + x'y' = 1$$

$$(x + y) \cdot (x'y') = 0$$

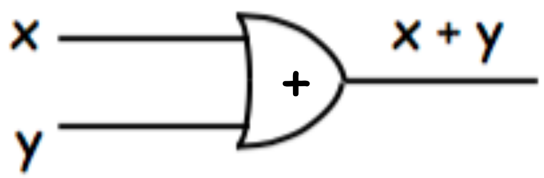
**HW 7.1 - assigned: If not done yet, continue by proving that the two equalities above hold!**

$x + y + x' y' = 1$
$(x + y) \cdot (x' y') = 0$

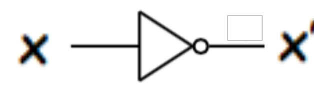
| | |
|---|---|
| $(x + y) \cdot 1 + x' y' =$ | 2 Identity |
| $= (x + y) \cdot (x + x') + x'y' =$ | |
| $= xx + xx' + yx + yx' + x'y' =$ | 7 Idempotant |
| $= (x + yx) + (xx') + (yx' + x'y') =$ | 12 Absorbtion, 5 Complement, |
| $= x + 0 + x'(y + y') =$ | 5 Complement, 2 Identity |
| $= x + x' \cdot 1 = x + x' = 1$ | |

| | |
|---|---|
| $(x + y) \cdot (x' y') =$ | |
| $= xx'y + yx'y' =$ | 5 Complement |
| $= 0 \cdot y + 0 \cdot x' = 0$ | 8 Identity, Annulment |

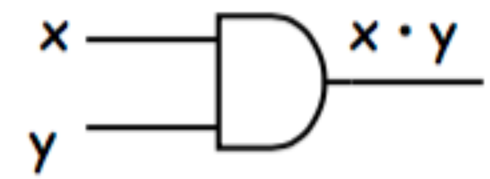
# Gates



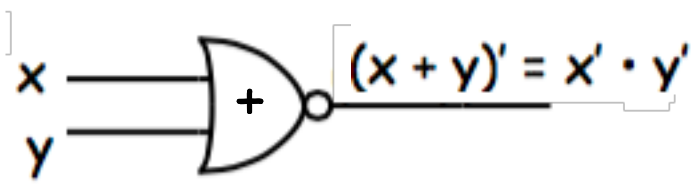

OR



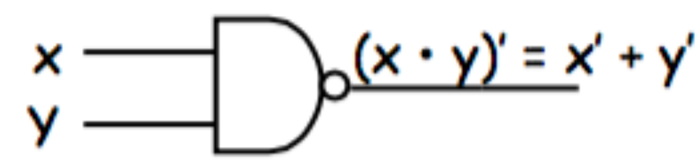

Inverter




AND




NOR

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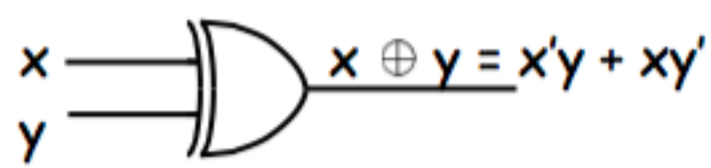
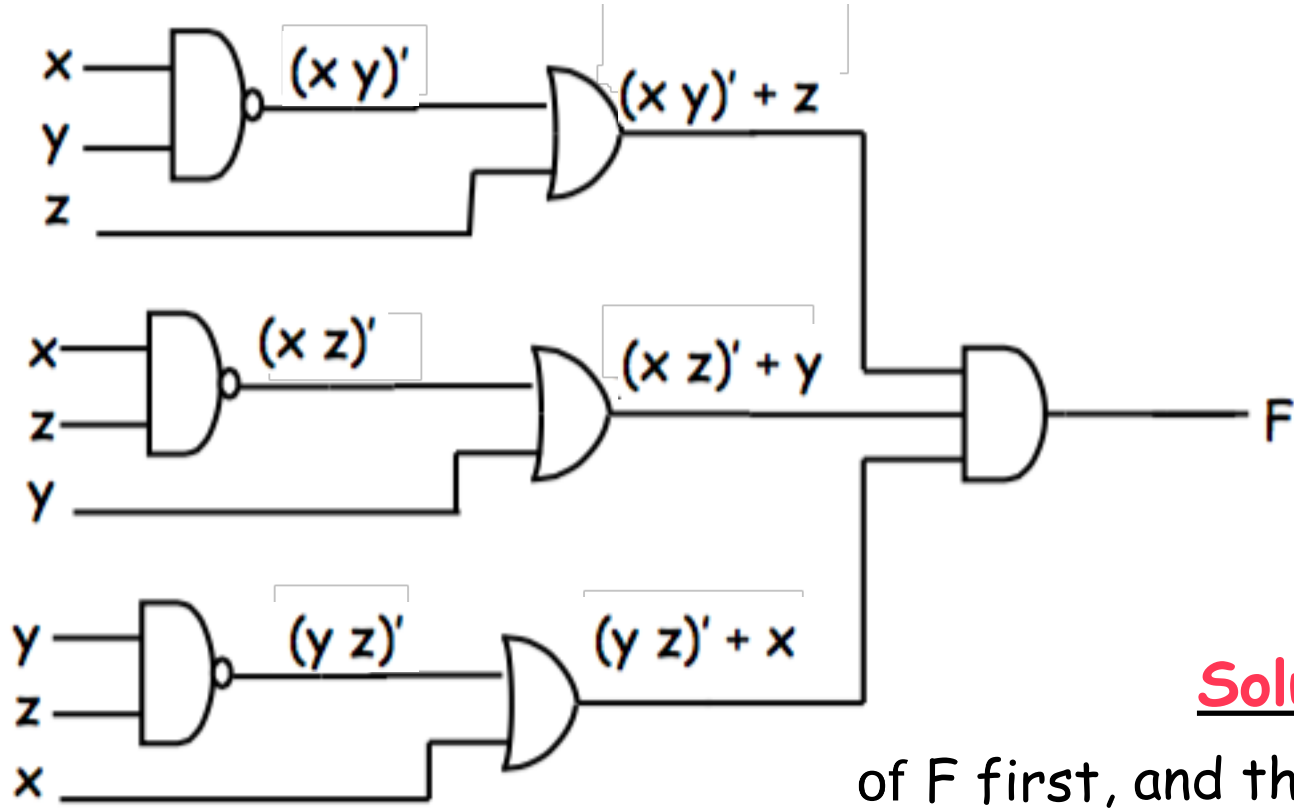



XOR
Exclusive OR

# Example – logic diagram




**Remember**:

AND:

| • | 0 | 1 |
|---|---|---|
| 0 | 0 | 0 |
| 1 | 0 | 1 |

OR:

| + | 0 | 1 |
|---|---|---|
| 0 | 0 | 1 |
| 1 | 1 | 1 |

**Problem**:

Find the truth table and expression of the function F.

**Solution** I recommend finding the expression of F first, and then deduce the truth table from the expression.

We have: $(xy)' + z = x' + y' + z$

$(xz)' + y = x' + z' + y$

$(yz)' + x = y' + z' + x$

What is then F?

$$F = (x' + y' + z)(x' + z' + y)(y' + z' + x)$$

What can we say about this product?

It is mostly = 0, like every product.

It is = 0 when any one of its factors = 0

$x' + y' + z = 0$ <--> $x'=0$ and $y'=0$ and $z=0$ <--> $x=1$ and $y=1$ and $z=0$

Similarly we have:

$x' + y + z' = 0$ <--> $x=1$ and $y=0$ and $z=1$

$x + y' + z' = 0$ <--> $x=0$ and $y=1$ and $z=1$

In rest, for all other values of x, y, z, F = 1

| x | y | z | F |
|---|---|---|---|
| 0 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 1 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 |
| 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 |

Truth table

HW 8.1

## Partial Solution + Hint

### (a) Prove: Inhibition is not commutative

Table gives two functions:

| | | | |
|---|---|---|---|
| $F_2 = xy'$ | $x/y$ | Inhibition | $x$, but not $y$ |
| $F_4 = x'y$ | $y/x$ | Inhibition | $y$, but not $x$ |

Let's choose $F_2 = x/y = x\,y'$

$F_2$ commutative $\overset{Det}{\Leftrightarrow}$ $F_2(x, y) = F_2(y, x)\ \ \forall x, y$ $\overset{Det}{\Leftrightarrow}$ $x/y = y/x\ \ \forall x, y$

$F_2$ not commutative $\overset{Det}{\Leftrightarrow}$ $\exists x, y$: $F_2(x, y) \neq F_2(y, x)$ $\overset{Det}{\Leftrightarrow}$ $\exists x, y$: $x/y \neq y/x$

In other words we need to find a counterexample to $x/y = y/x$, that is find values in {0, 1} for x, y, such that we have $x/y \neq y/x$ $\overset{Det}{\Leftrightarrow}$ $xy' \neq yx'$ as every B.A. has 0 and 1 as elements!

Make one side 1 and at the same time the other side 0.

$x = 1$ and $y = 0$ imply:

LS = $1 \cdot 1 = 1$ RS = $0 \cdot 0 = 0$ --> LS ≠ RS --> **F₂ is not commutative**

**Done**

**(b) Prove: Exclusive OR (XOR) is associative** Table:

| | | | |
|---|---|---|---|
| $F_6 = xy' + x'y$ | $x \oplus y$ | Exclusive-OR | $x$ or $y$, but not both |

$F_6(x, y) = x y' + x' y = x \oplus y$

$F_6$ associative $\overset{\text{Det}}{\Leftrightarrow}$ $(x \oplus y) \oplus z = x \oplus (y \oplus z)$ $\forall$ x, y, z

$\overset{\text{Det}}{\Leftrightarrow}$ $F_6(F_6(x,y),z) = F_6(x, F_6(y,z))$ $\forall$ x, y, z

We need to prove this for all BAs, so we need to use the axioms and properties of a BA.

$LS = (x y' + x' y) \oplus z = (x y' + x' y) z' + (x y' + x' y)' z =$

$= xy'z' + x'yz' + (x' + y)(x + y')z = xy'z' + x'yz' + x'y'z + xyz$ 5 Complement

$RS = x \oplus (yz' + y'z) = x(yz' + y'z)' + x'(yz' + y'z) = x(y' + z)(y + z') + x'yz' + x'y'z =$

$= xy'z' + xyz + x'yz' + x'y'z$ --> LS = RS --> **$F_6$ is associative**

**Done**

**HW 7.3** (continuation from class 8)

**2-9 b)**

$$\overline{((\bar{v}w+x)y+\bar{z})} = ((v+\bar{w})\cdot\bar{x}+\bar{y})z =$$

$$(v+\bar{w})\cdot\bar{x} = v\bar{x}+\bar{w}\bar{x}$$

$$= (v\bar{x}+\bar{w}\bar{x}+\bar{y})z = v\bar{x}z+\bar{w}\bar{x}z+\bar{y}z$$

**c)**

$$\overline{(wx(\bar{y}z+y\bar{z})+\bar{w}\bar{x}(\bar{y}+z)(y+\bar{z}))} =$$

$$=(\bar{w}+\bar{x}+(y+\bar{z})(\bar{y}+z))(w+x+y\bar{z}+\bar{y}z) =$$

$$=(\bar{w}+\bar{x}+yz+\bar{y}\bar{z})(w+x+y\bar{z}+\bar{y}z) =$$

$$=\bar{w}x+\bar{w}y\bar{z}+\bar{w}\bar{y}z+\bar{x}w+\bar{x}y\bar{z}+\bar{x}\bar{y}z+wyz+xyz+w\bar{y}\bar{z}+x\bar{y}\bar{z}$$

HW 7.3 (continuation)

2-9 d) $\overline{(A+\bar{B}+C)(\bar{A}\bar{B}+C)(A+\bar{B}\bar{C})} =$

$= \bar{A}B\bar{C} + (A+B)\cdot\bar{C} + \bar{A}\cdot(B+C) \overset{(4)}{=}$

$= \bar{A}B\bar{C} + A\bar{C} + B\bar{C} + \bar{A}B + \bar{A}C \overset{(12)}{=}$

$= A\bar{C} + B\bar{C} + \bar{A}B + \bar{A}C$

What now? Expand and then contract again:

$= AB\bar{C} + A\bar{B}\bar{C} + \cancel{AB\bar{C}} + \bar{A}B\bar{C} + \bar{A}B\bar{C} + \cancel{\bar{A}B\bar{C}} +$

$+ \cancel{\bar{A}BC} + \bar{A}\bar{B}C = A\bar{B}\bar{C} + AB\bar{C} + \bar{A}B\bar{C} +$

$+ \bar{A}BC + \bar{A}\bar{B}C = \boxed{A\bar{C} + \bar{A}B + \bar{A}C}$