# Karnaugh Maps

## CLASS 11

**HW 12** 12-A Given the Boolean functions $F_1$ and $F_2$,

(a) Show that the Boolean function $E = F_1 + F_2$ contains the sum of the minterms of $F_1$ and $F_2$.

## Solution

Let $F_1$, $F_2$ be two arbitrary B. functions expressed in canonical sum of products (minterms) form. Let's take first an arbitrary example to see what the situation is. It's not a proof, but gives us a hint.

Example: $F_1 = xyz + x'yz + x'y'z + x'y'z'$ $F_2 = xyz + x'yz + xy'z$

$E = F_1 + F_2 = xyz + x'yz + x'yz' + x'y'z' + xyz + x'yz + xy'z = \boxed{xyz + x'yz + x'y'z + x'y'z' + xy'z}$

**General case:** E will contain all the minterms in $F_1$ "or-"ed (added) to all the minterms of $F_2$. The only "simplification" will happen with minterms that are common to $F_1$ and $F_2$, which will appear only once (by (7)).

(b) Show that the Boolean function $G = F_1 F_2$ contains only the minterms that are common to $F_1$ and $F_2$.

## Solution

For (b) let's again consider the same example in the same way.

$G = F_1 \cdot F_2 = (xyz+x'yz+x'y'z+x'y'z')(xyz+x'yz+xy'z) = \boxed{xyz + x'yz}$

**General case:** G will contain only the minterms that are common to $F_1$ and $F_2$ **because**:

1) The product of identical minterms gives the same minterm as a result (by (7)).

2) If two minterms are different, then they must differ in at least one variable which will appear in its true value in one term, and in its complement in the other. In the product of such different minterms, after possibly commuting variables, we must have the product $var \cdot \overline{var}$, which by (5) = 0, and thus we have:

The product of two different minterms is 0.

HW 12 - continued

12-B List the truth table of the function:

$$F = xy + xy' + y'z$$

**Solution**

**Note**: No minterms!

Let's give the entries in the truth table corresponding to the 2-literal terms.
We can expand every 2-literal term to minterms, to identify the triplets in the truth table.

For instance $xy = xyz + xyz'$

The remaining triplets all correspond to $F = 0$

| x | y | z | F |
|---|---|---|---|
| 0 | 0 | 0 | 0 |
| 0 | 0 | 1 | 1 |
| 0 | 1 | 0 | 0 |
| 0 | 1 | 1 | 0 |
| 1 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 |
| 1 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 |

$y'z = 1$ (rows 001 to 101)

$xy' = 1$ (rows 100, 101)

$xy = 1$ (rows 110, 111)

Done

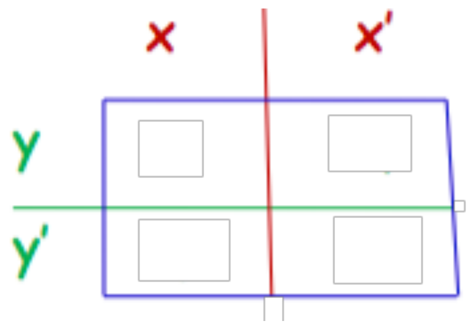
# Traditional Minimization Methods

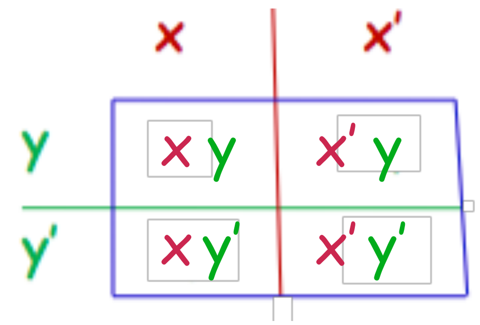
1) Karnaugh map
2) Tabulation method

**Karnaugh map**: Venn diagram like. **We visualize the function and the simplifications!**

**2 variables**: x, y - viewed as sets

**Note**: Each possible minterm has a position on the K map.
-> Have a region for each possible intersection.

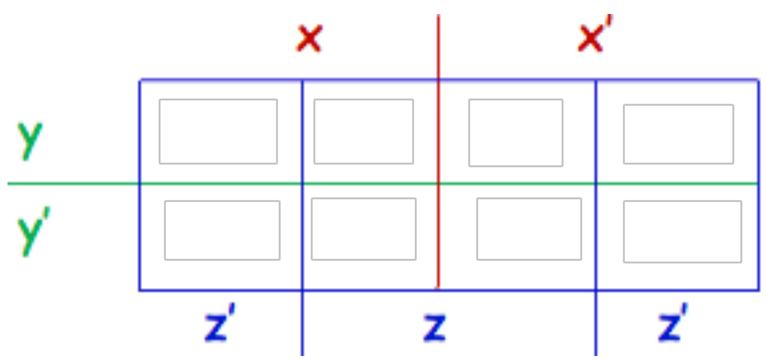


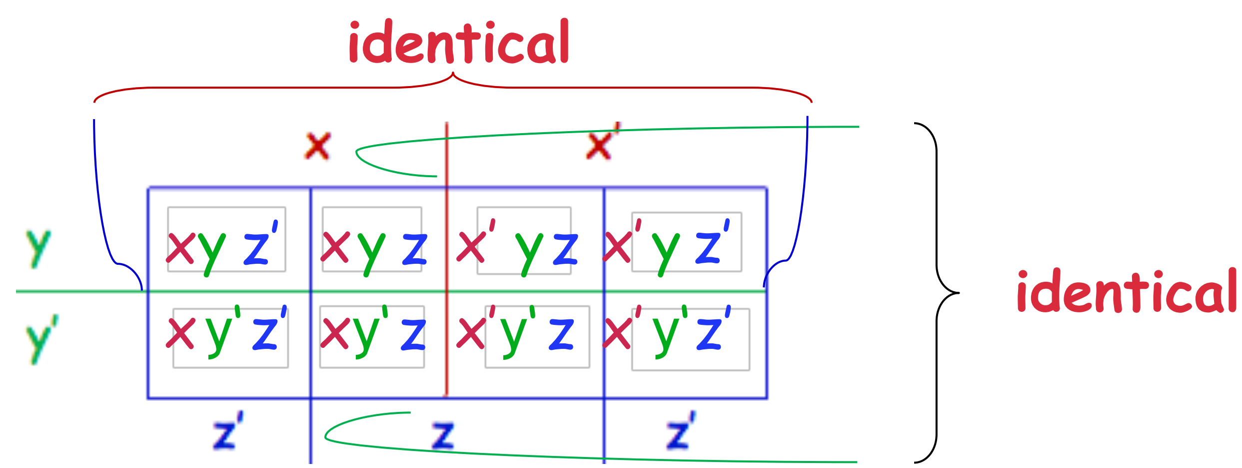




**NOTE:**

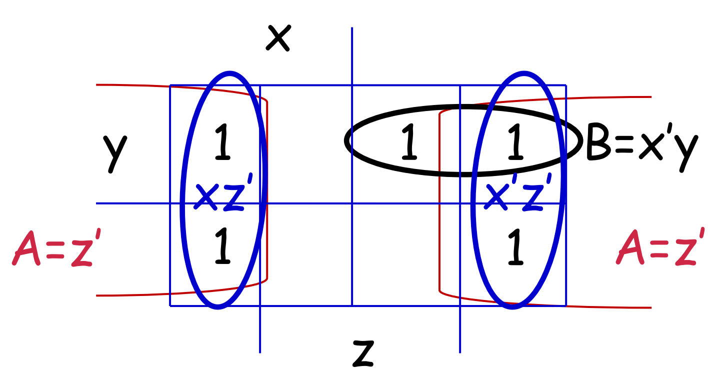
For the purpose of being adjacent these two sets of edges are considered **identical**:

**3 variables**: x, y, z - also sets




**Example**: f = xyz' + xy'z' + x'yz + x'yz' + x'y'z'




Put a 1 in the place of each minterm, as the value of f.

By putting together two adjacent squares we form a size-2 implicant.

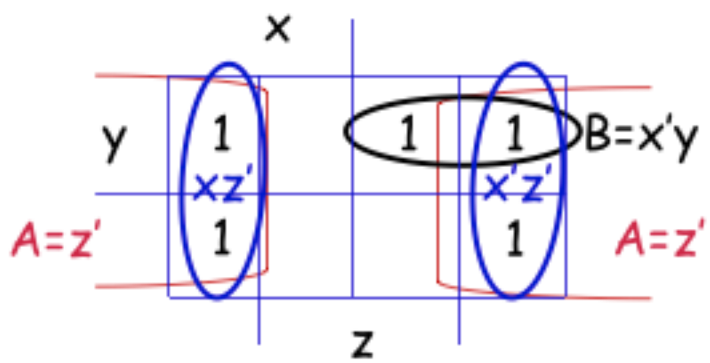
By putting together two adjacent size-k implicants we form a size-2k implicant.

This way we can only form implicants of size $2^k$, for some k.

The larger the size of an implicant, the fewer literals it contains. A **prime implicant** is maximal relative to set inclusion, that is it is an implicant that is not included ( $\subseteq$ ) in another implicant.

To obtain a minimal form we want to **form prime implicants only**. Here A, B only are prime implicants

From previous page we have:




**Goal**: Cover all 1's with the least # of prime implicants, i.e., largest possible implicants.

We get: $f = A + B$ in terms of sum of implicants.

We now replace each prime implicant by its expression and we obtain:

The minimal form: $f = z' + x'y$

## K map procedure

0) Put the 1's on the K map.
1) Find all prime implicants
2) Find all essential implicants

A prime implicant, I, is **essential**, whenever it contains a " 1 " that is only covered by implicant I.

**Note**: **The essential implicants must be in (all) the minimal form(s).**

3) **Purpose of Procedure:** **Cover all 1's with the least number of prime (largest) implicants.**

4) Write the minimal form.

You may want to print this:

**K map procedure**

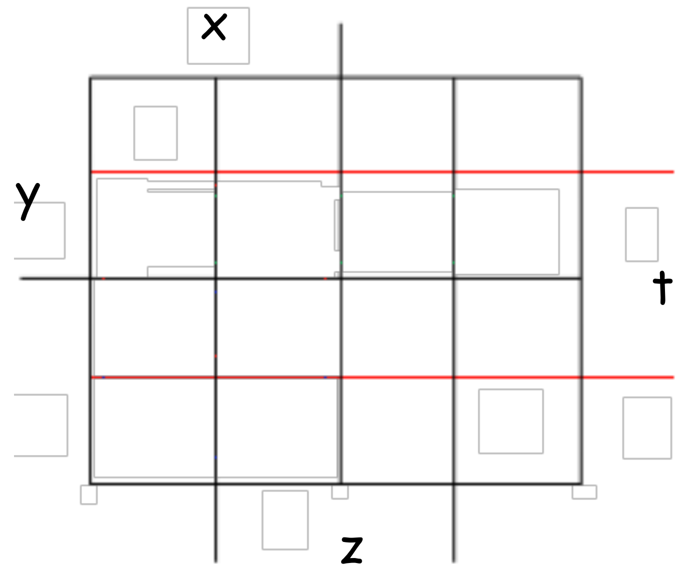
0) Put the 1's on the K map.
1) Find all prime implicants
2) Find all essentials (implicants)

**All essentials are in every minimal form.**

3) **Cover all 1's with the least number of prime (largest) implicants.**
4) Write the minimal form.

## 4 variables

Same idea:




**Example**:

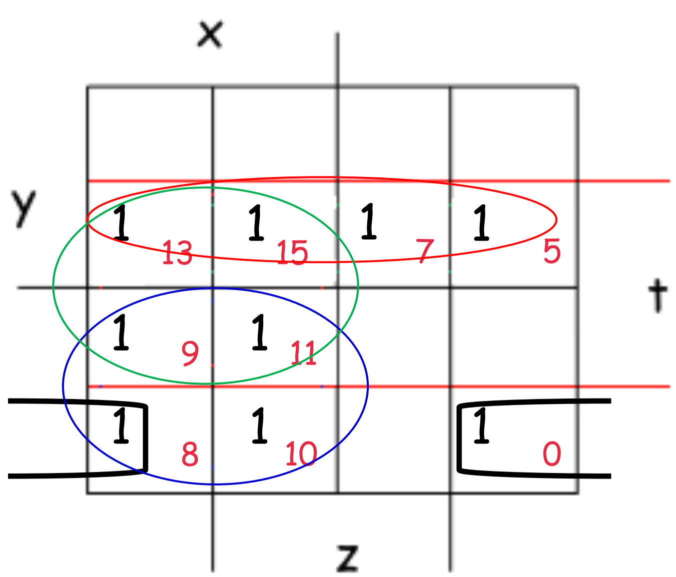
$f = \sum (0, 5, 7, 8, 9, 10, 11, 13, 15)$ = Give all minimal forms.

[ Let's write the decimal numbers in binary and then as minterms to put them on the K map:

Base 2: 0000, 0101, 0111, 1000, 1001, 1010, 1011, 1101, 1111 correspond to:

x'y'z't', x'yz't, x'yzt, . . . . . . . . . , xyzt ]

= x'y'z't' + x'yz't + x'yzt + xy'z't' + xy'z't + xy'zt' + xy'zt + xyz't + xyzt




Put the 1's on the K map.

**HW 13.1** Finish this: Using the procedure give **all minimal forms**. We'll just mark all the prime implicants now:

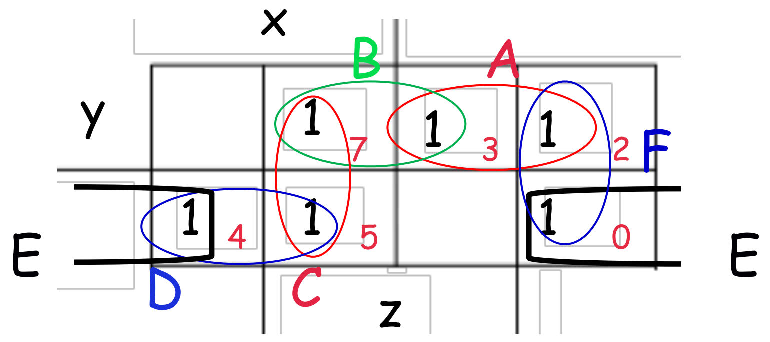
**HW 13.2** Give **all minimal forms** for the function:

$f = \sum (1, 5, 7, 8, 9, 10, 11, 14, 15)$

**Example**: function with several minimal forms

$f = \sum(0, 2, 3, 4, 5, 7)$

**Solution**




Prime implicants: A - F

Essential: none.

How can we cover all 1's with the least # of implicants?

We need at least 3 implicants in the minimal form to cover all 1's (there are 6 of them)

f = A + C + E = $x'y + xz + y'z'$

f = B + D + F = $yz + xy' + x'z'$

We have two minimal forms.