CL=CSCI 160

CLASS 2

<u>HW 1.1 (Remainder method-fractions)-Solution + Binary-to Hex conversion justification</u>

<u>Remainder Method – Fractions</u>

Let's use 4-bit precision. This works for all bases!

Ex.: 0.627 in base 10. What should we do?

<u>Solution: Multiply only the fraction part by 2 and record the integer part (which will either be 1 or 0).</u>

<u>Repeat until you reach the desired precision:</u>

.627 * 2 = 1.254

.254 * 2 = 0.508

.508 * 2 = 1.016

.016 * 2 = 0. . . .

0. 1 0 1 0

$0.627 \times 2 = 1.254$

$0.254 \times 2 = 0.508$

$0.508 \times 2 = 1.016$

$0.016 \times 2 = 0. \ldots$

$\boxed{0.1010}$

Conversion from base 2 <----> base 16

Justification

$379_{10} = 2^8 + 2^6 + 2^5 + 2^4 + 2^3 + 2^1 + 2^0$

1 0 1 1 1 1 0 1 1

Now remember: $16 = 2^4$

$$= 1\,\underbrace{(2^4)^2}_{16^2} + \underbrace{(2^4)^1}_{16^1}\underbrace{(2^2 + 2^1 + 2^0)}_{7} + \underbrace{(2^4)^0}_{16^0}\underbrace{(2^3 + 2^1 + 2^0)}_{11 = B}$$

$= 1\ 7\ B_{hex}$

HW 1.2 (Binary & Hex addition and subtraction)-Solution

Binary:

Note: Calculations are done w. 6-bit precision. Results may differ if a different precision is used!

Addition:

1 1 1 1 1 1 1 1 1 1 1 1 1

M = 3892.74_{10} 1111 0011 0100 . 1011 11 00 +

N = 9341.65_{10} 0010 0100 0111 1101 . 1010 01 00

11 0011 1011 0010 . 0110 00

Subtraction:

Decimal:

1 1 1

1324 -

999

325

Binary:

1 1 1 1 1 1 1 11 1

10 0100 0111 1101. 1010 01 -

1111 0011 0100. 1011 11

1 0101 0100 1000. 1110 10

Next will be **Hexadecimal**. Let's prepare for the grouping:

$M = 3892.74_{10}$ 1111 0011 0100.1011 11 00

$N = 9341.65_{10}$ 0010 0100 0111 1101.1010 01 00

$A = 10_{10} = 1010_2$

$B = 11_{10} = 1011_2$

$C = 12_{10} = 1100_2$

$D = 13_{10} = 1101_2$

$E = 14_{10} = 1110_2$

$F = 15_{10} = 1111_2$

Hexadecimal

Hex Addition:

```
     1  1 1  1
N =  247D.A4  +
M =   F34.BC
     -------
     33B2.60
```

$4+C=4+12=16=10_{hex}$

$1+A+B=[1+10+(5]+6)=16+6= 16_{hex}$

⋮

Compare to: 11 0011 1011 0010.0110 00 from previous page:

SAME!

Hex Subtraction:

```
     1   1  1
N =  247D.A4  -
M =   F34.BC
     -------
     1548.E8
```

$14_{hex} - C=16+4-C=20-12= 8_{hex}$

$1A_{hex} -1-B=16+10-1-B=25-11=14= E_{hex}$

⋮

Compare to: 1 0101 01 00 10 00. 111 0 10 from previous page:

SAME!

HW 1-A

Which is the largest binary number that can be expressed with 15 bits?
What are the equivalent decimal and hexadecimal numbers?

Solution:

$$\underbrace{111.....1}_{15}{}_2 = 1\,\underbrace{00.....0}_{15} - 1 = 2^{15} - 1$$

HW 1-B

Consider a system that contains 32K bytes. Assume we are using byte addressing, that is assume that each byte will need to have its own address, and therefore we will need 32K different addresses. For convenience, all addresses will have the same number ***n***, of bits, and ***n*** should be as small as possible.
What is the value of ***n*** ?

Solution:

32K bytes will need 32K addresses.

To make it more manageable, let's consider only 32 bytes: we need 32 smallest different numbers to express their addresses. They are 0 through 31, or in binary, they are 00000 to 11111, that is we need 5 bits to express these addresses, as $32 = 2^5$

Back to 32K bytes, we have $32K = 2^5 \times 2^{10} = 2^{15}$, that is we need 15 bits to express these addresses.

HW 1-C

The numbers in each of the following equalities are all expressed in the same base, ***r***. Determine this radix ***r*** in each case for the following operations to be correct.
(a) 14/2 = 5
(b) 54/4 = 13

Solution:

(a) 14/2 = 5 , all in base r; it implies $r \geq 6$ (just an observation!). Why?

We have LS $= 14_r / 2 = (r + 4) / 2$ RS = 5

We have: LS = RS, which implies: $r + 4 = 2 \times 5 = 10$ It follows $\boxed{r = 6}$

Proof: 14/2 = 5 , all in base r = 6. We convert to decimal:

LS $= 14_6 / 2 = (6 + 4)_{10}/2 = 10/2 = 5 =$ RS

(b) 54/4 = 13 , all in base r. LS $= (5r + 4)/4$ RS $= r + 3$

$5r + 4 = 4(r + 3)$ It follows $\boxed{r = 8}$

Proof: LS $= 54_8 / 4 = (44)_{10}/4 = 11_{10}$ RS $= 8 + 3 = 13_8 = 11_{10}$

SAME!

New HW assigned:

1–12. Perform the following binary multiplications:

(a) 1101×1011 **(b)** 0101×1010 **(c)** 100111×011011

1–14. A limited number system uses base 12. There are at most four integer digits. The weights of the digits are 12^3, 12^2, 12, and 1. Special names are given to the weights as follows: $12 = 1$ dozen, $12^2 = 1$ gross, and $12^3 = 1$ great gross.

(a) How many beverage cans are in 6 great gross + 8 gross + 7 dozen + 4?
(b) Find the representation in base 12 for 7569_{10} beverage cans.

1–16. *In each of the following cases, determine the radix r:
(a) $(BEE)_r = (2699)_{10}$ **(b)** $(365)_r = (194)_{10}$