CL=CSCI 160

CLASS 3

GIVE ANSWERS IN THE SPACES INDICATED BELOW.
JUSTIFY YOUR ANSWERS!
DO WELL!!!

1. Consider the following binary numbers:

   $M = 101101$
   $N = 11.001$

   Using 3-bit precision compute in binary:

1) MN
   Solution

```
          1 0 1 1 0 1 ×
              1 1 . 0 0 1
   -----------------------
 1 1 1 1 1 1 1
          1 0 1 1 0 1
      1 0 1 1 0 1
    1 0 1 1 0 1
   -----------------------
 1 0 0 0 1 1 0 0 . 1 0 1
```

2) M/N $M = 101101$ 3-bit precision

Solution $N = 11.001$

$$
\begin{array}{r}
1110.011 \\
11001 \,\big)\, 101101000.000 \\
11001 \\
\hline
101000 \\
11001 \\
\hline
011110 \\
11001 \\
\hline
0101000 \\
11001 \\
\hline
0011110 \\
11001 \\
\hline
\end{array}
$$

2. We have the following equality:

$24_r + 18_r = 40_r$

where all the numbers are written in the same base r, as indicated. Determine r, and clearly encircle the result.

Solution

Convert to decimal:

$2r + 4 + r + 8 = 4r$ <-> $\boxed{r = 12}$

Proof

Prove: $24_{12} + 18_{12} = 40_{12}$, that is:

$2x12 + 4 + 1x12 + 8 = 4x12$, which is equivalent to:

$28 + 20 = 48$ $\boxed{\text{True!}}$

1–12. Perform the following binary multiplications:

(a) 1101 × 1011 **(b)** 0101 × 1010 **(c)** 100111 × 011011

(a)

```
          1101 ×
          1011
  1 1 1  1
   -----------
          1101
         1101
       1101
   -----------
    10001111
```

(c)

```
                 100111 ×
                  11011
          1
    1 1 1 1    1 1 1 1
         -----------------
                 100111
                100111
              100111
             100111
   ------------------------
          10000011101
```

1–14. A limited number system uses base 12. There are at most four integer digits. The weights of the digits are 12^3, 12^2, 12, and 1. Special names are given to the weights as follows: 12 = 1 dozen, 12^2 = 1 gross, and 12^3 = 1 great gross.

(a) How many beverage cans are in 6 great gross + 8 gross + 7 dozen + 4?
(b) Find the representation in base 12 for 7569_{10} beverage cans.

$12^2 = 144 \; ; \; 12^3 = 1728$

(a) $6 \times \overbrace{1728}^{=12^3} + 8 \times \overbrace{144}^{=12^2} + 7 \times 12 + 4 = \ldots$

(b) $7569_{10} = 4 \;\overbrace{\text{gr. gross}}^{1728} + 4 \;\overbrace{\text{gross}}^{144} + 6 \;\overbrace{\text{doz}}^{12} + 9 =$

$$\begin{array}{r} 7569 \\ 6912 \\ \hline =657 \\ 576 \\ \hline =81 \\ 72 \\ \hline 9 \end{array}$$

$= \boxed{4\;4\;6\;9_{\text{base }12}}$

1–16. *In each of the following cases, determine the radix r:

(a) $(BEE)_r = (2699)_{10}$ **(b)** $(365)_r = (194)_{10}$

(a) $\underbrace{B}_{11} \cdot r^2 + \underbrace{E}_{14} \cdot r + \underbrace{E}_{14} = 2699$

$Br^2 + Er = 2699 - 14 = 2685$

alternative 1 (Note: r is a divisor of 2685: factorize it!)

$r(Br + E) = 2685 = 179 \times \underbrace{3 \times 5}_{\text{base}}$ $15 = r$

alternative 2 (Use quadratic formula)

$11r^2 + 14r - 2685 = 0$ Use formula to solve quadratic equations

to get the result, but the coefficients are large! $r_{1,2} = \begin{cases} 15 \\ -\text{something} \end{cases}$

(b) Solve : $3r^2 + 6r + \underbrace{5 - 194}_{-189} = 0$ Use same formula to solve quadratic

equations to get the result: Result $\boxed{r = 7}$

(r-1)'s Complement-Representation

r = base   Very important: How are our numbers stored?

$$N = \underbrace{\overbrace{\qquad\qquad}^{\text{integer part}}}_{n} \cdot \underbrace{\overbrace{\qquad\qquad}^{\text{fraction part}}}_{m}$$

n, m = number of locations

Definition

r = 2   (N is a binary number!)

How did you compute N_{1c} = 1's complement of N? By swapping 0 <-> 1, which is equivalent to:

$$N_{1c} = \underbrace{1 \ldots\ldots 1}_{n}.\underbrace{1 \ldots\ldots 1}_{m} - N = 1\underbrace{0 \ldots\ldots 0}_{n}.\underbrace{0 \ldots\ldots 0}_{m} - 0.\underbrace{0 \ldots\ldots 01}_{m} - N = 2^{n} - 2^{-m} - N$$

Base r:

$$\boxed{N_{(r-1)c} = r^{n} - r^{-m} - N}$$

Algorithm for subtracting two numbers using only addition and the (r-1)'s complement

To perform $\boxed{M - N}$ do:

1) $M + N_{(r-1)c}$

2)

a) If there is an e. a. c. (= end-around-carry = overflow), then add it to the l. s. d. (= least significant digit) of result from 1). Stop.

b) If there is no e. a. c., then the result is negative, and is obtained by taking the (r-1)'s complement of what we obtained at 1); in other words, we compute:

$$-\underbrace{(M + N_{(r-1)c})}_{\text{Result from 1)}}{}_{(r-1)c}$$

Stop.

Example in binary + HW assigned

Alg (M-N): 1) $M + N_{(r-1)c}$ 2) a) e.a.c. -> add it to l.s.d.
b) no e.a.c. -> compute -(r-1)'s compl. of 1)

Ex-binary r = 2 Suppose n = 4, m = 2

2-a) Path: i)

Suppose: M: 1101.10
N: 1011.01

Apply algorithm to get M - N:

$$\begin{array}{lr} & {}^{1\ 1\ \ 11} \\ 1)\ M: & 1101.10 \ + \\ N_{1C}: & 0100.10 \\ \hline \text{e.a.c.} = 1 & 0010.00 \ + \\ & 1 \\ \hline & \boxed{0010.01} \end{array}$$

->case2-a)

2-b) Path: ii) Swap M <-> N, to get to case 2-b)

M: 1011.01
N: 1101.10

Apply algorithm to get M - N:

$$\begin{array}{lr} & {}^{1\ \ \ 1} \\ 1)\ M: & 1011.01 \ + \\ N_{1C}: & 0010.01 \\ \hline & 1101.10 \end{array}$$

no e.a.c. ->case2-b)

-> -1's compl.: $\boxed{-\ 0010.01}$ as expected

HW (more to follow):

Why does this algorithm work?

Why does this algorithm work?

Hint

integer part   fraction part

We know: $N = \underbrace{\quad\quad\quad}_{n} \cdot \underbrace{\quad\quad\quad}_{m}$

$$\boxed{N_{(r-1)c} = r^n - r^{-m} - N}$$

1) $M + N_{(r-1)c} = M + r^n - r^{-m} - N$

2) Whether we follow a) or b) depends on the presence of an e. a. c.

What is the magnitude of the e.a.c.?   It's an 'overflow':

$\boxed{\text{e.a.c}}$: $\underbrace{10 \ldots\ldots 0}_{n}$ $\boxed{= r^n}$ On **branch a)** there is an e. a. c., which means we have:

$\underbrace{M + r^n - r^{-m} - N}_{\text{from 1)}} \geq \underbrace{r^n}_{\text{e.a.c.}}$ <--> $M - N \geq r^{-m}$ <-->

=smallest positive number in our representation

<--> $M - N > 0$ or $M > N$ It also means, that the case when

$M - N = 0$ will take branch b), which means that 0 will be expressed as -0 by this Alg.

Continue justifying the computations in the branches 2-a) and 2-b) as HW.

3-A Do the following conversion problems:

(a) Convert decimal 34.4375 to binary.

(b) Calculate the binary equivalent of $1/3$ out to 8 places. Then convert from binary to decimal. How close is the result to $1/3$?

(c) Convert the binary result in (b) into hexadecimal. Then convert the result to decimal. Is the answer the same?

3-B **Determine the value of base x if $(211)_x = (152)_8$.**

3-C Noting that $3^2 = 9$, formulate a simple procedure for converting base-3 numbers directly to base-9. Use the procedure to convert $(2110201102220112)_3$ to base 9.

3-D The solutions to the quadratic equation

$x^2 - 11x + 22 = 0$

are $x = 3$ and $x = 6$.

Determine the base of the numbers in the equation.

3-E Convert the hexadecimal number 68BE to binary, and then convert it from binary to octal and then to base 32.